Efficient time-step coupling for hybrid continuum/ molecular modelling of unsteady micro-scale gas flows

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Abstract

In this paper we describe a numerical method for the efficient time-accurate coupling of hybrid continuum/molecular micro gas flow solvers. Hybrid approaches are commonly used when non-equilibrium effects in the flow field are spatially localized; in these regions a more accurate, but typically more expensive, solution procedure is adopted. Although this can greatly increase efficiency in steady flows, in unsteady flows the evolution of the solution as a whole is restricted by the maximum time step allowed by the molecular-based/kinetic model; numerically speaking, this is a stiff problem. In the method presented in this paper we exploit time-scale separation, when it exists, to partially decouple the temporal evolution of the two parts of the hybrid model. This affords major computational savings. The method is a modified/extended version of the seamless heterogeneous multiscale method (SHMM). Our approach allows multiple micro steps (molecular steps) before coupling with the macro (continuum) solver: we call this a multi-step SHMM. This maintains the main advantages of SHMM (computational speed-up and flexible application) while improving on accuracy and greatly reducing the number of continuum computations and instances of coupling required. The improved accuracy of the multi-step SHMM is demonstrated for two canonical one-dimensional transient flows (oscillatory Poiseuille and oscillatory Couette flow) and for rarefied-gas oscillatory Poiseuille flow.